Integrable systems in perturbative quantum field theory

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ENIGMA 08

Largely based on

- 0805.1197 [hep-th]
  (with Christian Schwinn)
- hep-th/0702035, hep-th/0604040
  (with David Skinner and Lionel Mason)
- 'the literature'

What am I selling today?

known

The self-dual Yang-Mills equations in 4 dimensions are integrable [Penrose and Ward]

main idea in our work

Full equations as a perturbation around self-dual sector [Mason, 05]

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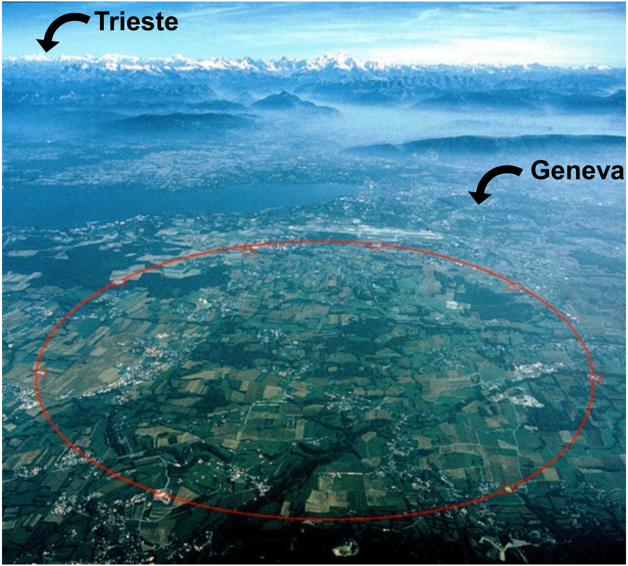
The big picture



Some definitions and terminology

- Standard model of particle physics is a Yang-Mills theory
- $\rightarrow$ Yang-Mills equations predict experimental outcomes

Pure Yang-Mills theory

ingredients:

- A the gauge field: a Lie-algebra valued connection 1-form of a fibre bundle over $\mathbb{R}^4$ (say $U(N)$)
- $F = dA + g(A \wedge A)$ its curvature 2 form
- g the coupling constant
- $S = \mathrm{Tr} \int F \wedge *F$ the Yang-Mills action
- $\frac{\partial S}{\partial A} = 0$ are the Yang-Mills equations
- equations non-linear if $g \neq 0$
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From actions to amplitudes

- want: solve Yang-Mills equations with plane-wave asymptotics.
- $\rightarrow$ scattering amplitudes in perturbative series in g
- well-defined procedure to calculate these in terms of path integrals (cf. math-ph/0204014)
- scattering data: Lorentz group rep (P_μ and polarization)
  - Hodge $*$ projects 2 forms onto $\pm$ 1 eigenvalues: self-dual and anti-selfdual.
  - (anti) self-dual connections A solve full Yang-Mills equations.
  - example: polarization of external fields (helicity)

surprise

some scattering amplitudes are very, very simple [80's]

- $A(++\ldots+) = 0$, $A(-+\ldots+) = 0$, $A(--+\ldots+) =$simple
- despite rapid growth ($\sim n^2$) of ordinary diagram complexity

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Theorem (Ward-Penrose correspondence [$\sim$ 75])

selfdual solutions to the Yang-Mills equations on $\mathbb{R}^4$ (S^4)
(on to one)
holomorphic flat vector bundles on $\mathbb{CP}^3$

- CP^3 is the projective twistor of four dimensional space
- Euclidean: locally, $CP^3 = R^4 \times \mathbb{CP}^1$
- (see [Woodhouse, 85]) ingredients: $(0,1)$ forms, Lie algebra valued connection a, weight 0, adjoint valued b, weight -4

in action terms

$$S = \mathrm{Tr} \int_{\mathbb{CP}^3} d\Omega \wedge b \wedge (\bar{\partial} a + g a \wedge a)$$

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Self-dual Yang-Mills theory as an integrable system

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- $4D$ field theory (known as self-dual YM) as an integrable system

puzzle:

- Space-time SD YM $\to$ one non-trivial scattering amplitude A(++-) (classically), while the twistor system is free

resolution:

- Scattering amplitude is part of the field transformation (cf. KdV) [Ettle-Fu-Fudger-Mansfield-Morris, 07]

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Full Yang-Mills

summary of knowledge up to 2003

$$\begin{array}{llll} \mathbb{CP}^3 & \int_{\mathbb{CP}^3} d\Omega \wedge b \wedge (\bar{\partial} a + g a \wedge a) & & + \; ? \\ \mathbb{R}^4 & \qquad \int_{\mathbb{R}^4} B \wedge F_+ & & \int_{\mathbb{R}^4} F \wedge * F \end{array}$$

- In dec. 2003 Witten proposed 'twistor string theory' (also [Nair, 88])
- Amplitudes from topological B-model on supertwistor space $\mathbb{CP}^{3|4}$ ('super Calabi-Yau')
- Very inspiring in the physical community ($\sim$ 400 citations)
- [Mason 2005] and our work provides missing term:

$$S_{\text{miss}} = \int_{\mathbb{R}^4 \times \mathbb{CP}^1 \times \mathbb{CP}^1} d^4x \wedge dk_1 \wedge dk_2 \wedge (HbH^{-1})_1 \wedge (HbH^{-1})_2 \langle 12 \rangle^2$$

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$$(\bar{\partial}_0 + a_0) H[a_0] = 0$$

$$S_{\text{miss}} \sim \sum_{i=0}^{\infty} \left(\sum_{j=1}^{i} ba \ldots b_j \ldots a \right)$$

- use same transformation to action angle coordinates as before
- interpolates between usual Yang-Mills eqns and 'CSW rules'
- generates those 'simple' scattering amplitudes mentioned before
- re-orders complicated perturbation theory
- $\rightarrow$ perturbation around an integrable system!

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Full Yang-Mills is integrable?

- twistor variables provide a transform to action-angle variables → trivializes 1 three particle amplitude
- Yang-Mills has two three particle amplitudes ($A(++-)$ and $A(+--)$)
- 'Other' three particle amplitude simplified on 'dual' twistor space
- Can you simplify both at the same time?

→ ambi-twistor space? [Mason, Skinner, 05]

BCFW recursion

Britto-Cachazo-Feng-Witten (05): every amplitude in Yang-Mills can be expressed in terms of certain sums over those 2 three particle amplitudes only.

- Is full Yang-Mills integrable?
- BCFW: reconstruction of scattering amplitudes through a field transformation?

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More interesting ideas floating around

- similar constructions for physically interesting cases
  - supersymmetry (very natural)
  - massive gauge theories
- beginning of understanding of the quantum aspects (much harder)
- similar construction for Einstein gravity [Mason, Skinner, 08]

BCFW remarks

- BCFW works in 10 dimensions [Arkhani-Hamed, Kaplan, 08]
- recursive BCFW structure is very natural in string theory [Boels, Larsen, Obers, Vonk, 08]
- relation to topological strings? CFT?

More interesting ideas floating around

- similar constructions for physically interesting cases
  - supersymmetry (very natural)
  - massive gauge theories
- beginning of understanding of the quantum aspects (much harder)
- similar construction for Einstein gravity [Mason, Skinner, 08]

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Conclusions / questions

- Interesting developments from/with physical motivation
- New input/life for twistor theory and ideas

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- Is full (supersymmetric?) Yang-Mills quantum mechanically integrable?
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